6.6 - Solving Exponential and Logarithmic Equations

Warmup

1. $\log_3 7 + \log_3 x = \log_3 14$

   2

2. $\log_2 10 - \log_2 t = \log_2 2$

   5

3. $\log_3 y - \log_3 2 = \log_3 12$

   24

4. $\log_3 14 + \log_3 m = \log_3 42$

   3

5. $\log_5 x = 3\log_5 7$

   343

6. $\log_2 p = \frac{1}{2}\log_2 81$

   9

7. $\log_9 x = \frac{1}{2}\log_9 144 - \frac{1}{3}\log_9 8$

   6

8. $\log_7 m = \frac{1}{3}\log_7 64 + \frac{1}{2}\log_7 121$

   44

Conics - Hyperbolas

Circle *Parabola* *Ellipse* *Hyperbola*

Conics - Parabola

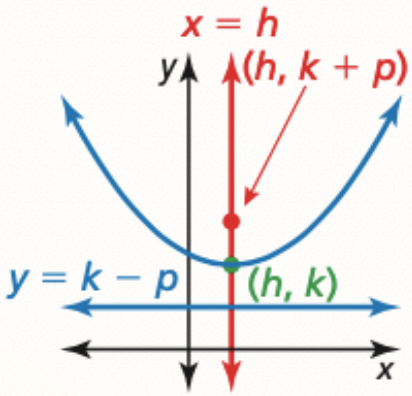
Standard Equations of a Parabola with Vertex at (h, k)

Vertical axis of symmetry ($x = h$)

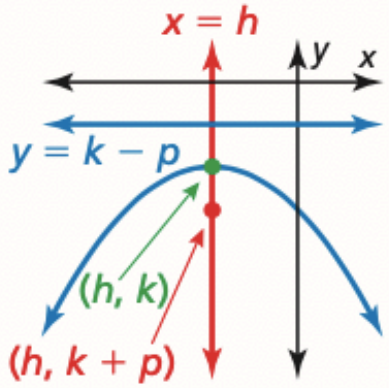
Equation: $y = \frac{1}{4p}(x - h)^2 + k$

Focus: $(h, k + p)$

Directrix: $y = k - p$



$p > 0$



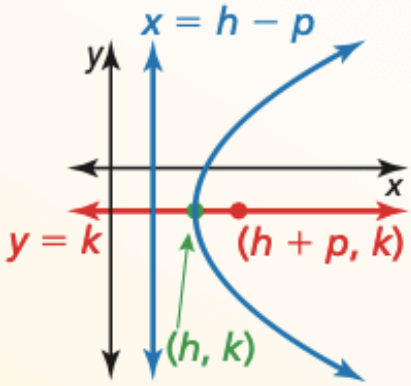
$p < 0$

Horizontal axis of symmetry ($y = k$)

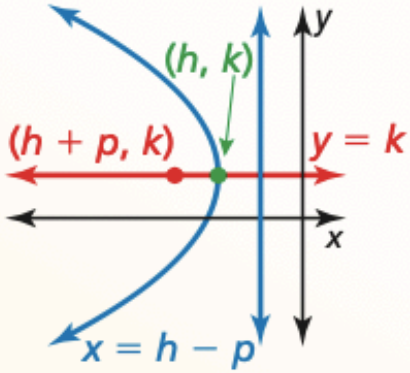
Equation: $x = \frac{1}{4p}(y - k)^2 + h$

Focus: $(h + p, k)$

Directrix: $x = h - p$



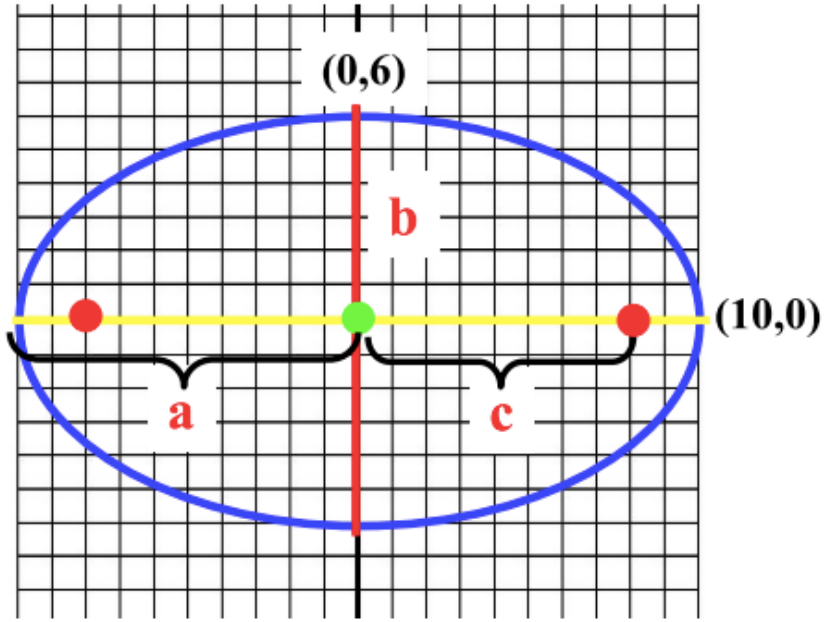
$p > 0$



$p < 0$

Conics - Ellipses

The equation for an ellipse.



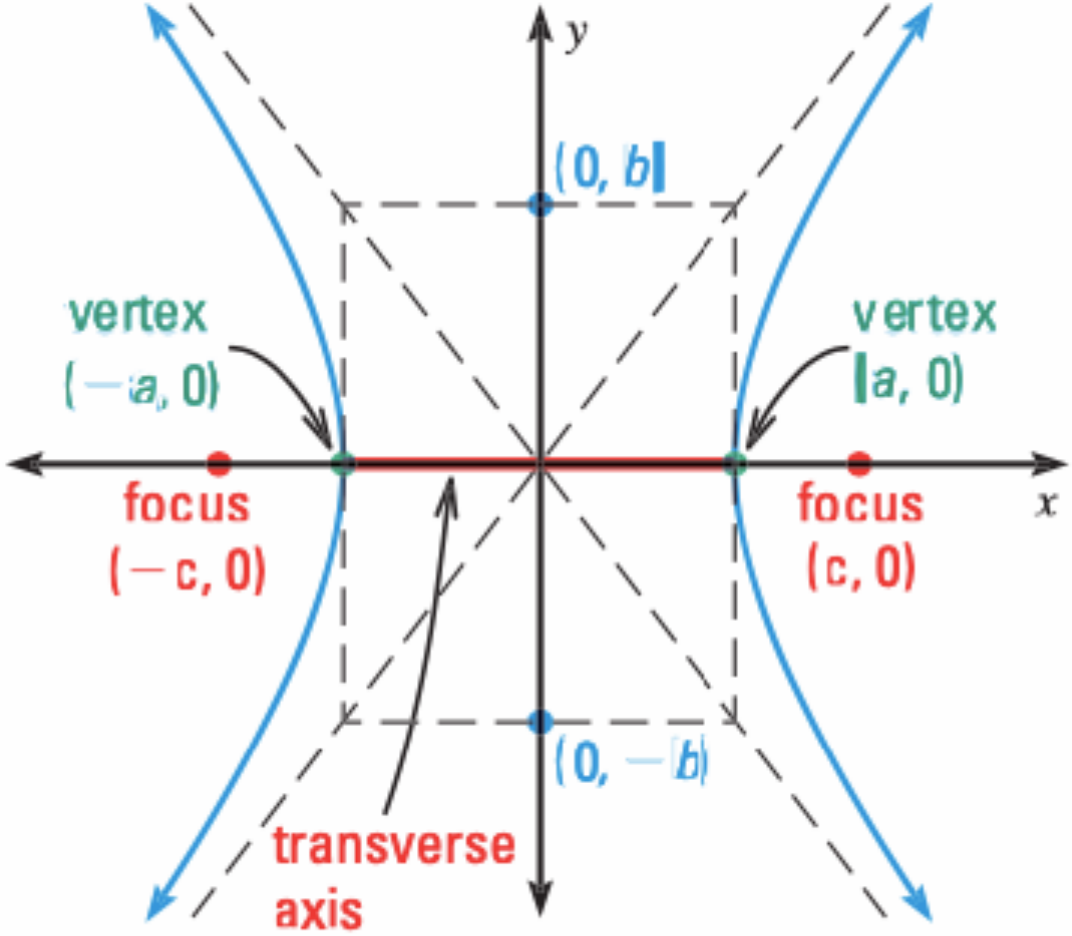
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{10^2} + \frac{y^2}{6^2} = 1$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

Conics - Hyperbola

Definition



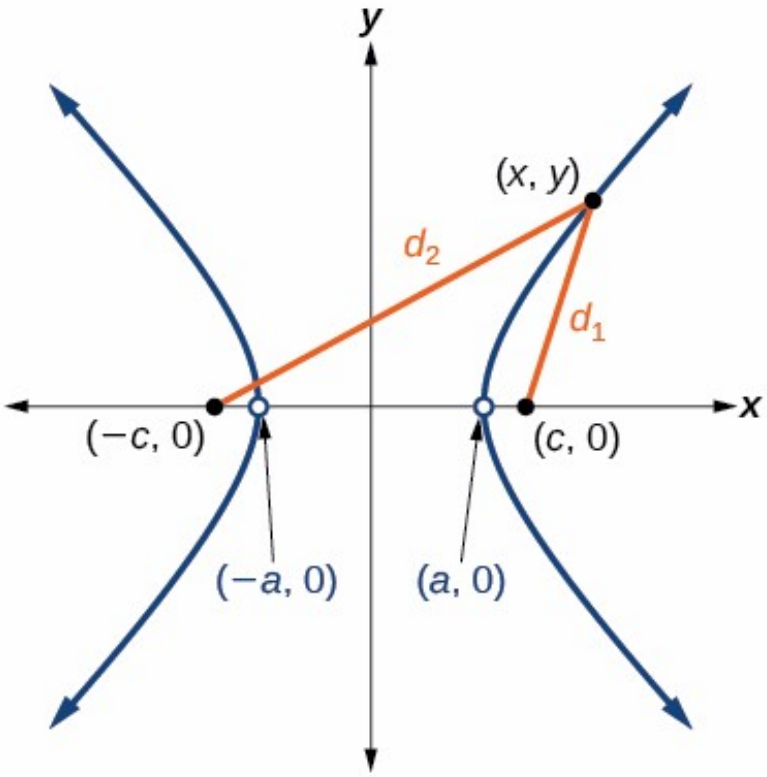
Hyperbola with horizontal transverse axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Conics - Hyperbola

The equation for an hyperbola.

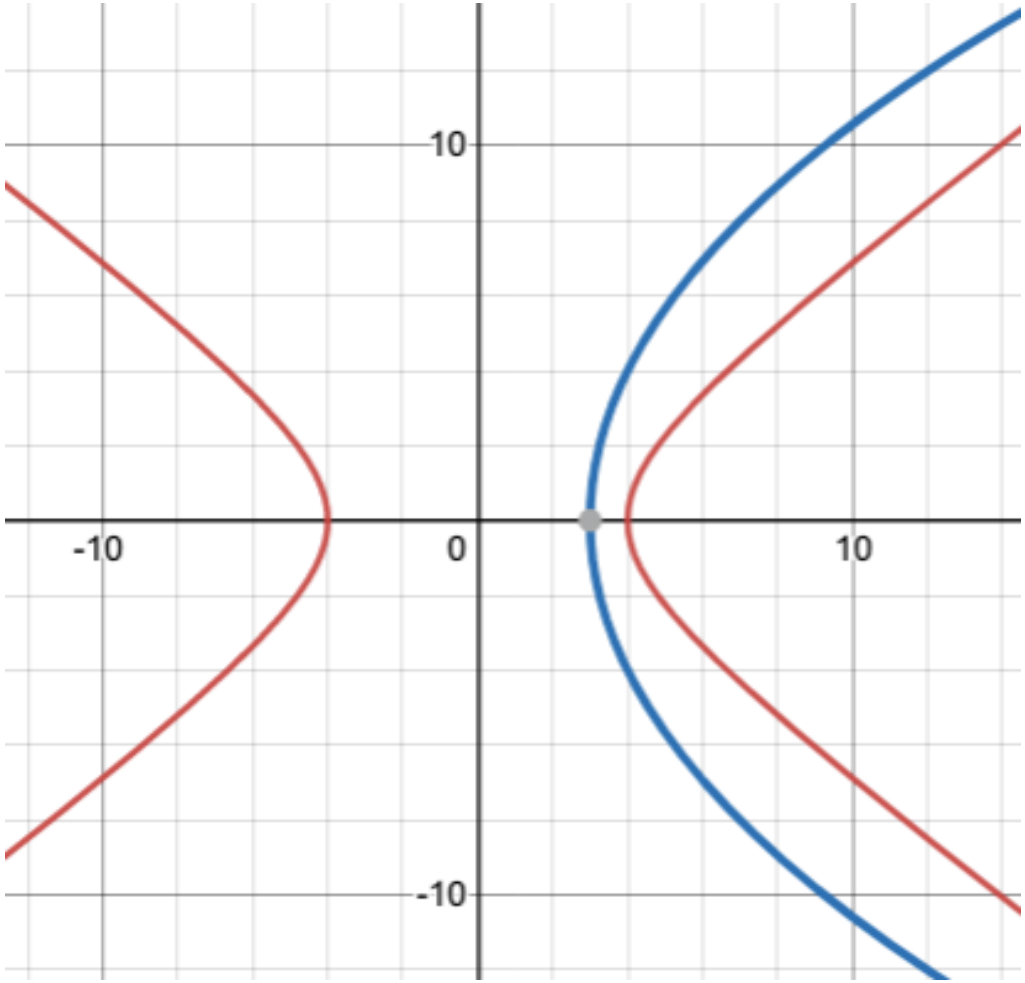
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



d2-d1 is constant

Conics - Hyperbola

Is a hyperbola the same as a parabola?



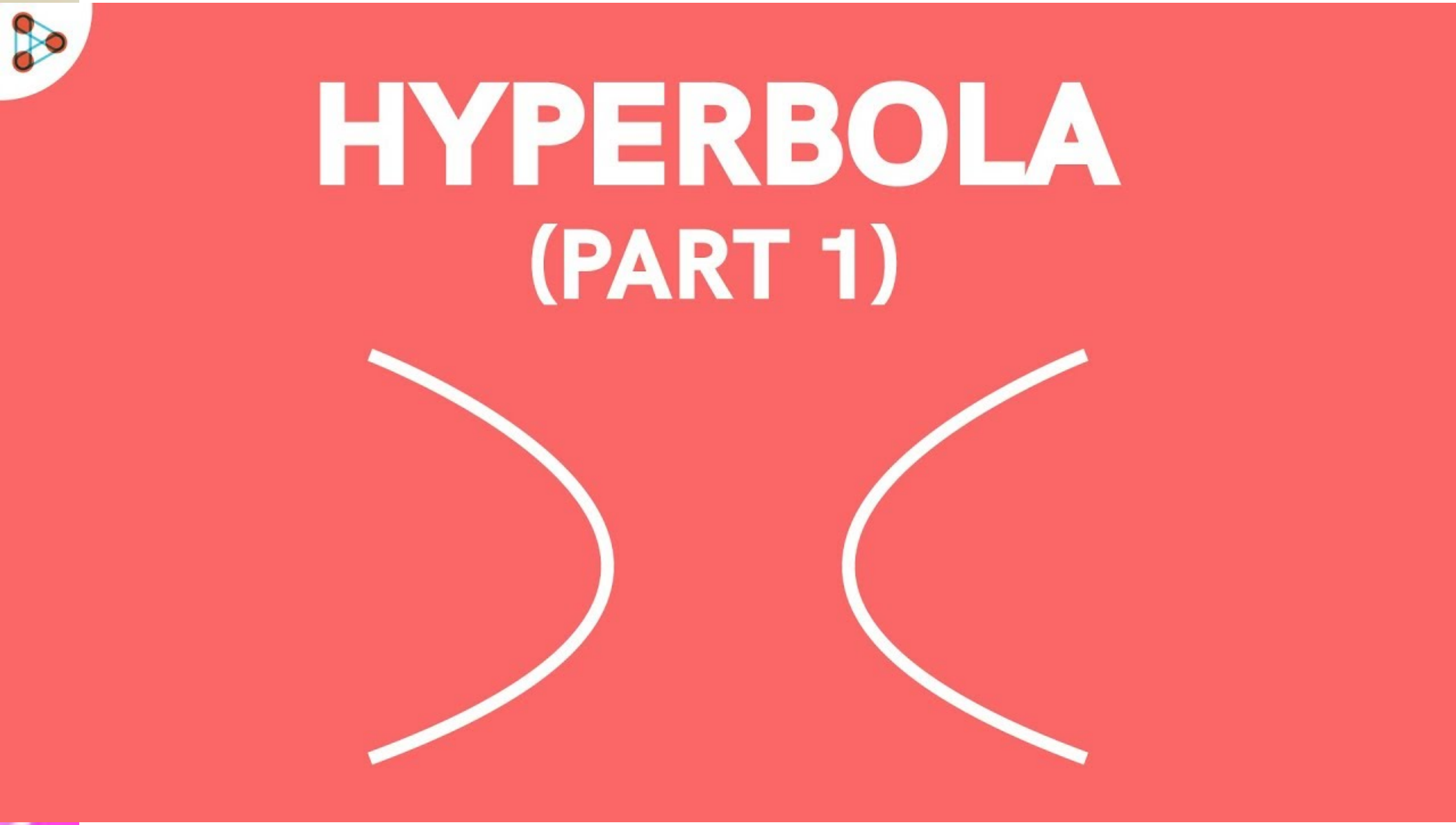
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$x = a(y-k)^2 + h$$

Conics - Hyperbolas

Start 3:15

Conics - Hyperbolas



Conics - Hyperbolas

Old

$$\frac{y^2}{16} - \frac{x^2}{9} = 1$$

New

$$4x^2 - y^2 = 16$$

$$\frac{4x^2 - y^2 = 16}{16} \qquad \frac{x^2}{4} - \frac{y^2}{16} = 1$$

Practice

1. $25x^2 - 4y^2 = 100$

$$\frac{x^2}{4} - \frac{y^2}{25} = 1$$

2. $-4x^2 = -9y^2 + 36$

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

Conics - Hyperbolas

General equation

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$4x^2 - y^2 + 24x + 10y - 5 = 0$$

$$\frac{(x+3)^2}{4} - \frac{(y-5)^2}{16} = 1$$

Practice

1. $9x^2 - 4y^2 - 18x - 16y = 43$

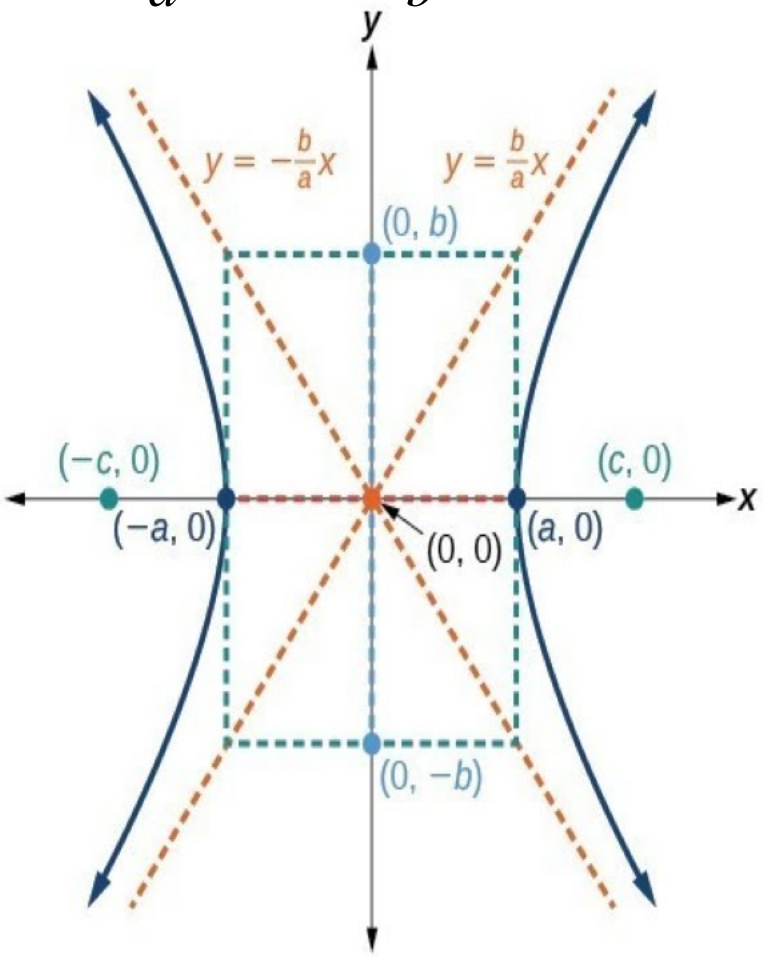
$$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1$$

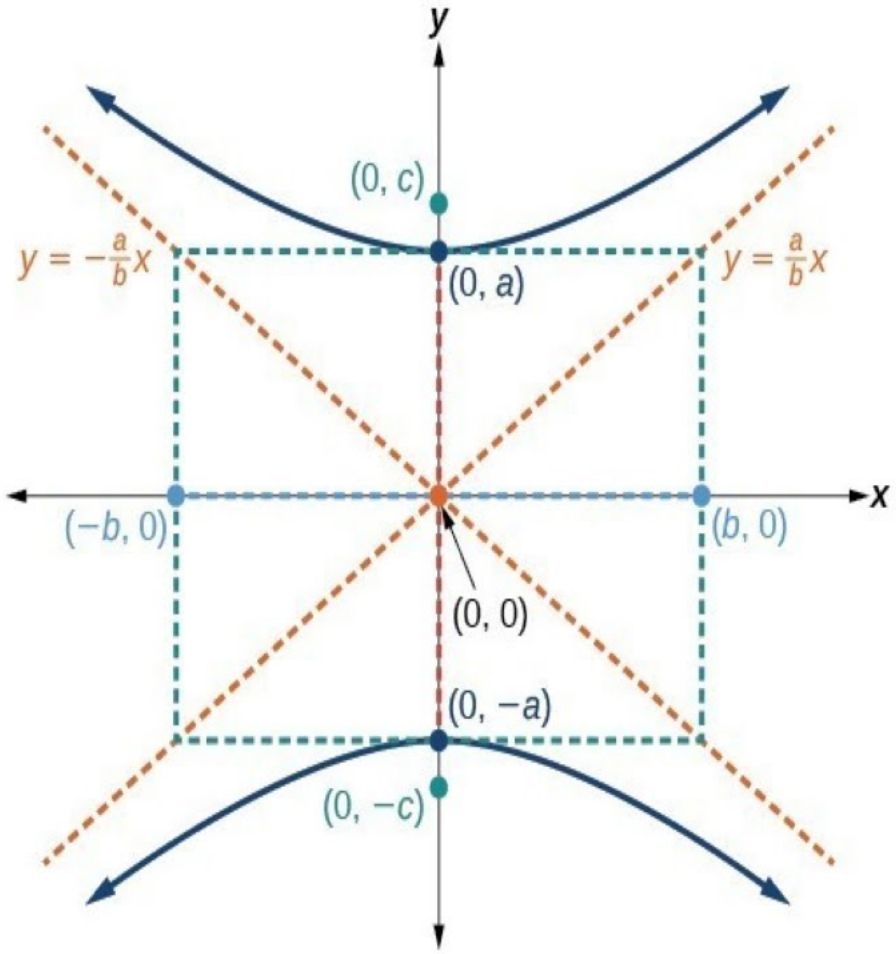
Conics - Hyperbolas

General equation

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$a^2 + b^2 = c^2$$

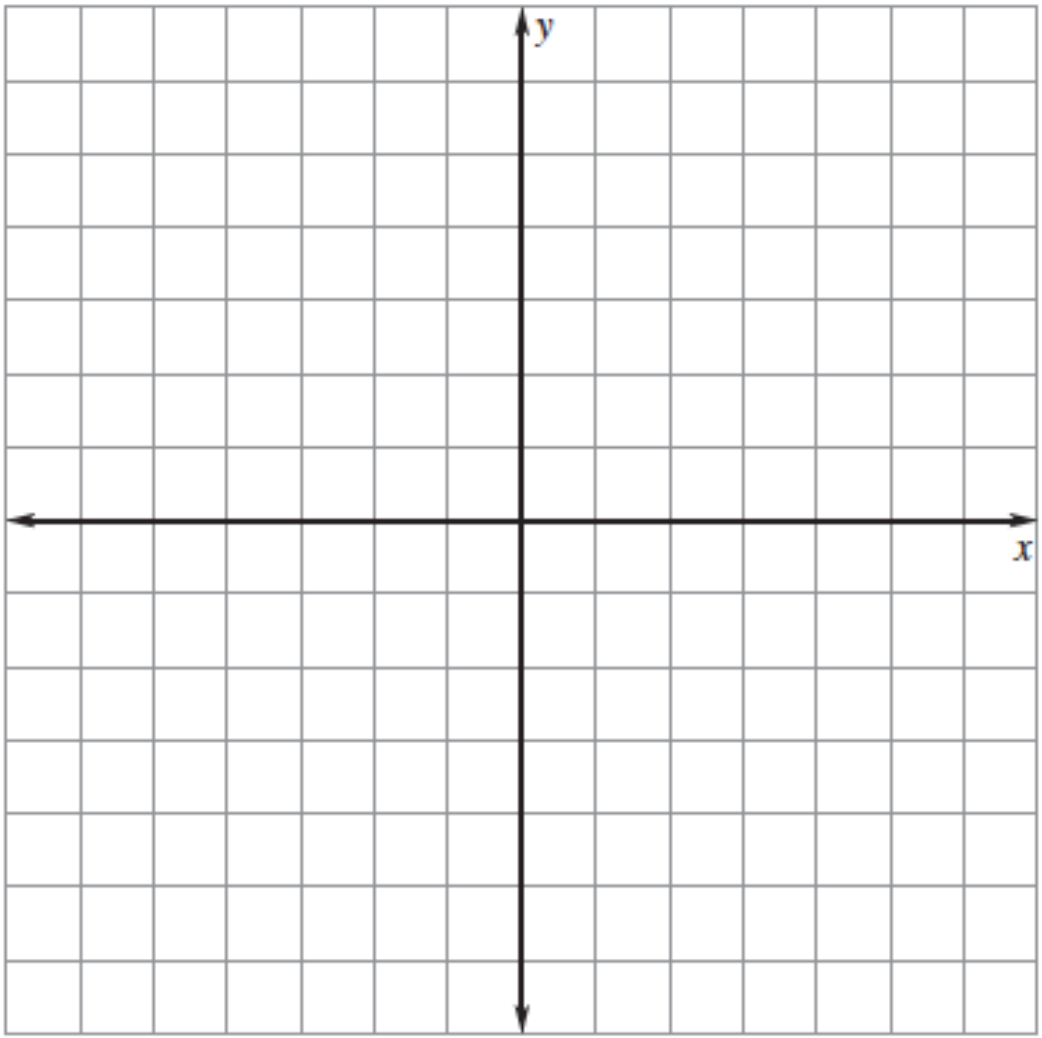




Conics - Hyperbolas

Let's graph

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$



Practice

$$\frac{y^2}{16} - \frac{x^2}{9} = 1$$

Conics - Hyperbolas

Find the equation:

$f_1(6,0),\ f_2(-6,0)$

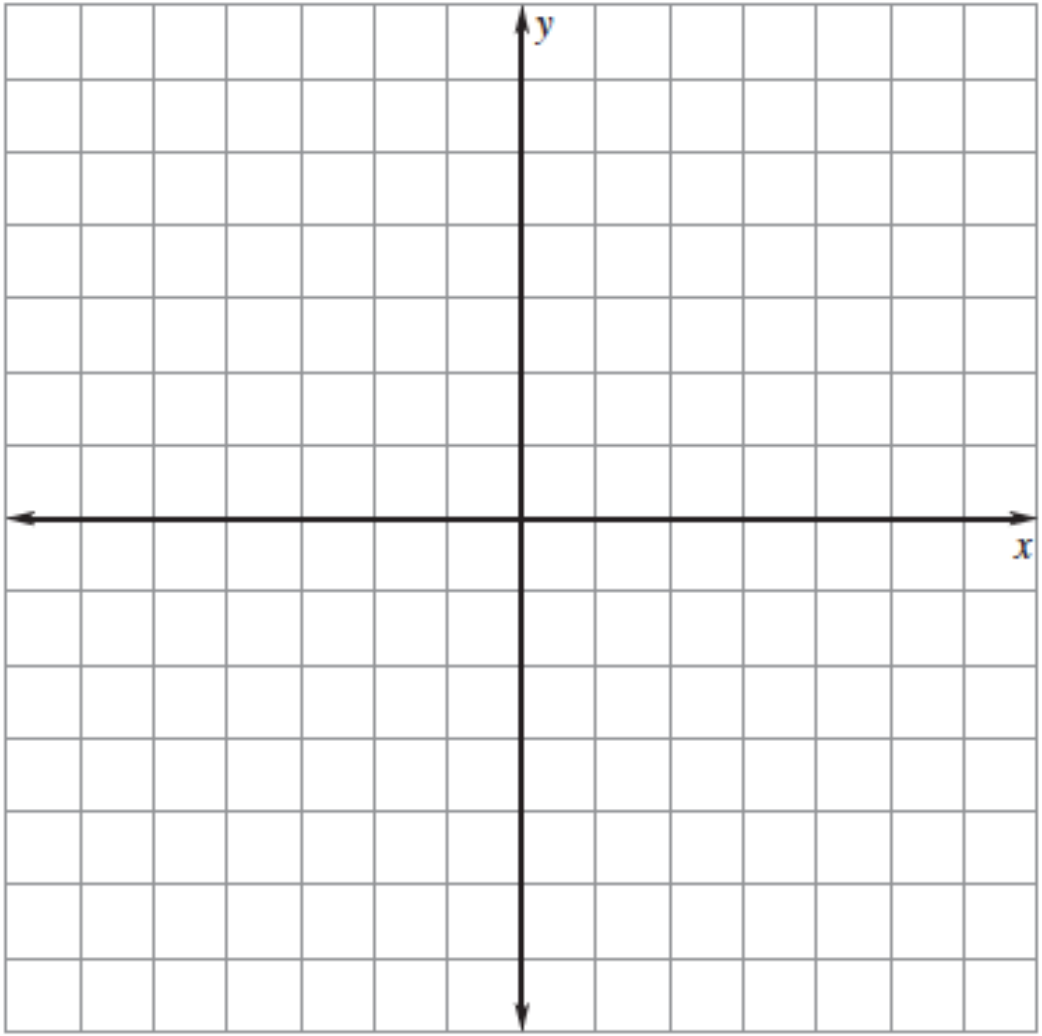
focal radii = 8

$c = 6$

$a = 4$

$b^2 = c^2 - a^2 = 20$

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$



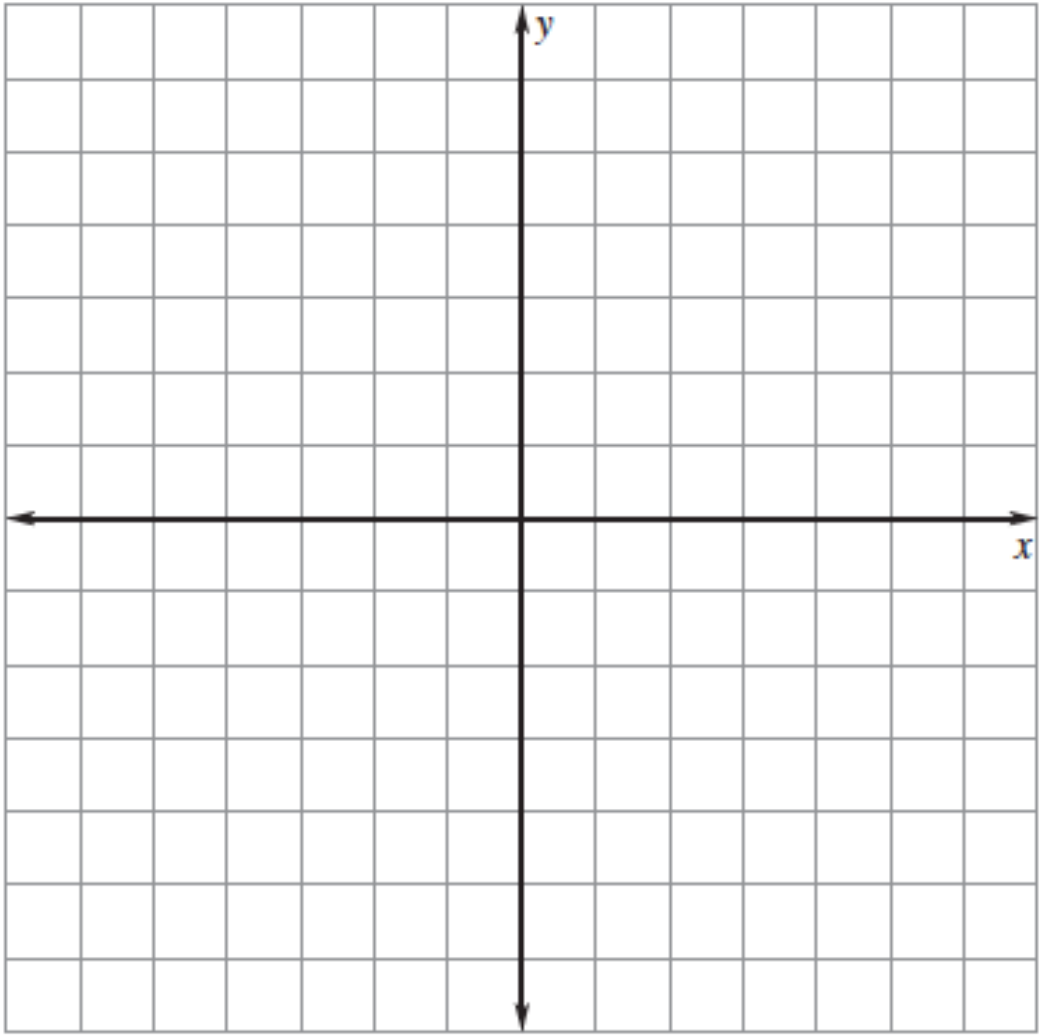
Conics - Hyperbolas

Change the focii:

$f_1(6,2),\ f_2(-6,2)$

focal radii = 8

$$\frac{x^2}{16} - \frac{y^2}{20} = 1?$$



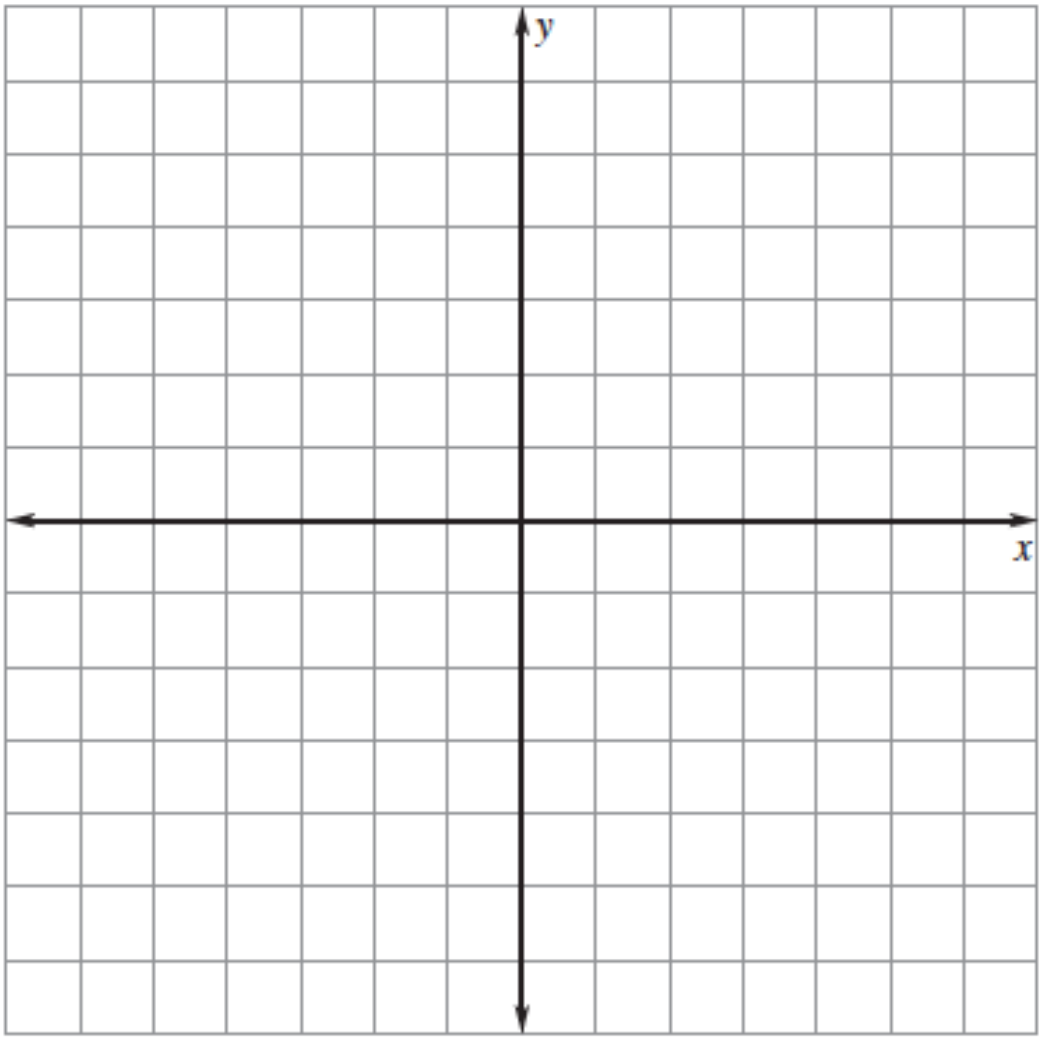
Conics - Hyperbolas

Practice

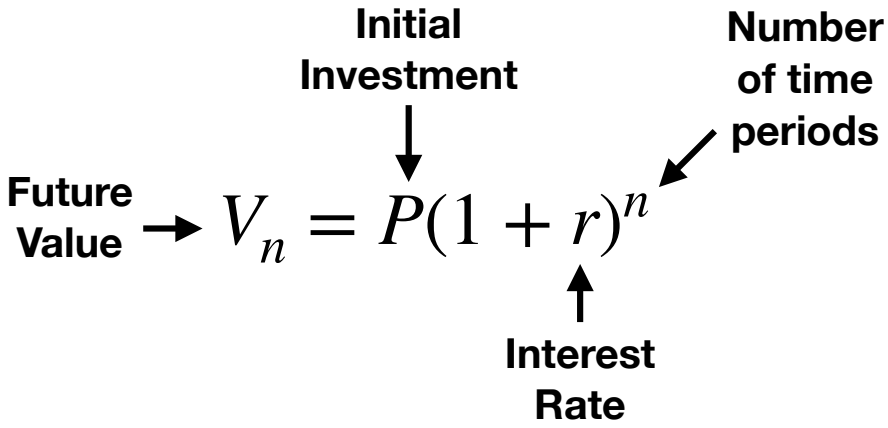
$f_1(0,8),\ f_2(0,-8)$

focal radii = 14

$$\frac{y^2}{49} - \frac{x^2}{15} = 1$$



Compounded Interest



An initial investment of $25,000 is made with a yearly interest rate of 5%.
What is the value after 6 years?

$$V_n = P(1 + r)^n$$
$$V_6 = 25000(1 + 0.05)^6$$
$$V_6 = 25000(1.05)^6$$
$$V_6 = 25000(1.34)$$
$$V_6 = 33502.39$$

What if it were compounded monthly?

$$V_6 = 25000\left(1 + \frac{0.05}{12}\right)^{6 \cdot 12}$$

33,725.44

Compounded Interest

Deposit $1,000 with a yearly interest rate of 5% compounded monthly.
What is the value after a year?

$$A(t) = A_0(1 + r)^t \qquad A(12) = 1000\left(1 + \frac{0.05}{12}\right)^{12} = 1{,}051.16$$

Practice

Invest $30,000 at 4.5% interest per year. What is the value after one year?

1) Interest is compounded monthly $\quad 30000\left(1 + \frac{0.045}{12}\right)^{12} = 31{,}378.19$

2) Interest is compounded weekly $\quad 30000\left(1 + \frac{0.045}{52}\right)^{52} = 31{,}380.23$
(1 yr = 52 wks)

3) Interest is compounded daily $\quad 30000\left(1 + \frac{0.045}{365}\right)^{365} = 31{,}380.75$
(1 yr = 365 days)

Compounded Interest

Euler's Constant - continuous interest

Invest $30,000 at 4.5% interest per year. What is the value after one year?

If compounded continuously?

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e=2.71828...$$

$$30000e^{0.045\cdot 1}=31{,}380.83$$

Practice

Invest $20,000 at 6.5% interest per year. What is the value after one year?

1) Interest is compounded monthly $20000\left(1+\frac{0.065}{12}\right)^{12}=21{,}339.44$

2) Interest is compounded weekly $20000\left(1+\frac{0.065}{52}\right)^{52}=21{,}342.31$

3) Interest is compounded continuously $20000e^{0.065\cdot 1}=21{,}343.18$